ation, and weight will descend with an acceleration $a-b$ times the angular acceleration.

**99. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

In a triangle $ABC$, base$=b$, area$=\triangle$, the principal moments of inertia at the centroid are $\frac{1}{72}m[a^2+b^2+c^2\pm\sqrt{(a^4+b^4+c^4-a^2b^2-a^2c^2-b^2c^2)}]$ and the principal axes at this point make with the base $AC$ an angle $\theta$ given by

$$\tan 2\theta=\frac{4(c^2-a^2)\triangle}{(a^2-c^2)^2-b^2(a^2+c^2)+2b^4}.$$

**Solution by the PROPOSER.**

Let $O$ be the centroid and transform from the rectangular axes $Ox$, $Oy$, to the oblique axes $Ox$, $OB$.

Also let $(b/6AD)(2AD-3y)=x'$.

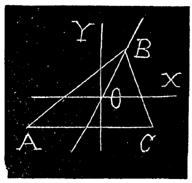
$$\text{Then } \Sigma mxy=\rho\sin^2 D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}(x+y\cos D)y\,dy\,dx=\tfrac{1}{36}b\rho AD^3\sin^2 D\cos D$$

$$=\tfrac{1}{18}mAD^2\sin D\cos D.$$

$$\Sigma mx^2=\rho\sin D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}(x+y\cos D)^2\,dy\,dx=\tfrac{1}{72}m(3b^2+4AD^2\cos^2 D).$$

$$\Sigma my^2=\rho\sin^3 D\int_{-\frac{1}{3}AD}^{\frac{2}{3}AD}\int_{-x'}^{x'}y^2\,dy\,dx=\tfrac{1}{18}mAD^2\sin^2 D.$$

$$\therefore\ \tan 2\theta=\frac{4AD^2\sin 2D}{3b^2+4AD^2\cos 2D}.$$




$$\text{But } \sin D=a\sin C/AD,\ \cos D=\frac{4AD^2+b^2-4a^2}{4b.AD}.$$

$$\sin 2D=\frac{a\sin C(4AD^2+b^2-4a^2}{2b.AD^2}=\frac{2(c^2-a^2)\triangle}{b^2.AD^2}.$$

$$\cos 2D=\frac{AD^2-2a^2\sin^2 C}{AD^2}=\frac{2(a^2-c^2)-2b^2(a^2+c^2)+b^4}{4b^2.AD^2}.$$

$$\therefore\ \tan 2\theta=\frac{4(c^2-a^2)\triangle}{(a^2-c^2)^2-b^2(a^2+c^2)+2b^4}.$$

$$A\cos^2\theta+B\sin^2\theta=\tfrac{1}{72}m(3b^2+4AD^2\cos^2 D).$$

$A\sin^2\theta+B\cos^2\theta=\frac{1}{18}mAD^2\sin^2D.$

$\therefore\ A+B=\frac{1}{72}m(3b^2+4AD^2).$

$(A-B)\cos2\theta=\frac{1}{72}m(3b^2+4AD^2\cos2D).$

$\therefore\ A=\frac{1}{72}m[a^2+b^2+c^2+2\surd(a^4+b^4+c^4-a^2b^2-a^2c^2-b^2c^2)].$

$B=\frac{1}{72}m[a^2+b^2+c^2-2\surd(a^4+b^4+c^4-a^2b^2-a^2c^2-b^2c^2)].$

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## MISCELLANEOUS.

**85. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.**

Prove that at least one of the three sides of a rational right triangle must be divisible by 5.

**II. Solution by M. A. GRUBER, A.M., War Department, Washington, D. C.; HON. JOSIAH H. DRUMMOND, LL. D., Portland, Me.; and the PROPOSER.**

Rational right triangles are divided into three kinds: (1), prime integral; (2), composite integral; and (3), fractional.

It is evident that prime integral right triangles are the basis of work.

To produce a prime integral right triangle we take *two integers prime to each other*, *one odd and the other even*. Then, twice their product will give one of the legs, the difference of their squares will give the other of the legs, and the sum of their squares will be the hypotenuse; or taking $p$ and $q$ as the two integers, the respective sides are $2pq$, $p^2-q^2$, and $p^2+q^2$.

This process will produce any prime integral right triangle.

The squares of even numbers end in 4, 6, and 0, and of odd numbers in 1, 9, and 5.

When the two prime integers, or $p$ and $q$ end, respectively, in 0 and an odd figure, or in 5 and an even figure, their product, or $pq$ of the side $2pq$ contains the factor 5; the other sides being odd numbers *not* ending in 5.

When the squares of the two prime integers, or $p^2$ and $q^2$ end, respeetively, in 1 and 6 or in 9 and 4, the difference of these squares, or the side $p^2$ and $q^2$ ends in 5, and is, therefore, divisible by 5.

When the squares of the two prime integers end, respectively, in 1 and 4 or in 9 and 6, the sum of these squares, or the side $p^2+q^2$ ends in 5.

The above contains all the combinations of any two prime integers, one odd and the other even, according to the formation of the sides of prime integral right triangles.

$\therefore$ In every *prime* integral right triangle one of the sides and *only one* is divisible by 5.

In composite integral and fractional right triangles either *one* or *all* of the sides must be divisible by 5. For, in order to have more than one side divisible by 5, the highest common factor of the three sides must contain the factor 5.

$\therefore$ In rational right triangles either *one* or *all* of the sides will be divisible by 5.